Extra Chapter: Multisided platforms

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[1]OpenTable is a website where diners can browse local restaurants and make reservations from their phone or computer. The website is easy to use, searchable by geographic location, type of food, or how expensive the restaurant is, and - importantly - diners can use the website for free! Often, diners can even earn bonus rewards for making multiple reservations using OpenTable. It makes the process of securing a restaurant reservation much simpler for the diner. But how can this business stay afloat if it charges its users a price of *zero dollars* (and in some cases, a price of *negative dollars* if rewards are earned) to participate?

We typically model a traditional firm as transforming inputs into an output, then selling that output to consumers. But OpenTable operates a bit differently. For starters, diners do not buy meals from OpenTable; rather, they use the site to allow them to more easily buy a meal from a restaurant! Similarly, restaurants who post open reservations are not looking to OpenTable to take up those reservations. They hope to use OpenTable to find potential diners to take them.

OpenTable is a **multisided platform**[2][3]: it is a platform which helps to facilitate transactions between different sides of the market for restaurant meals. Buyers (the diners) and sellers (the restaurants) can always seek each other out without OpenTable (a diner can call and make an individual restaurant reservation directly), but the platform makes the process easier for each party. Diners have the advantage of browsing many options, looking at menus, prices, and user reviews - all in one digital location. Restaurants know that by posting openings on OpenTable, the openings are likely to be seen by more potential diners. Because they seek to connect two or more separate

[1]This chapter is meant as a late-in-the-semester chapter, and therefore assumes basics of firm theory, including profit maximization, market power, and (potentially) some measures of market power have been discussed.

[2]**multisided platform**: a model of a firm where the firm facilitates transactions between different sides of a market.

[3]**two-sided platform**: a special kind of multisided platform where the firm facilitates transactions between two sides of a market, often in the form of a buyers' side and sellers' side.

parties in a market, multisided platforms are often called **matchmakers**.

Platforms have existed in the economy for quite sometime. Shopping malls and farmers' markets (which provide a centralized location to connect shoppers and retailers), real estate agents (who seek to connect home buyers with home sellers), and credit cards (which provide a service to facilitate transactions between buyers and merchants) are all platforms. However, in the last 20 years, platforms have played an increasingly prominent role in the economy. In fact, the 5 largest global corporations - Amazon, Apple, Facebook, Google, and Microsoft - all use platforms as a primary component of their business model:

- Amazon.com is an online marketplace which serves as a platform to connect sellers and buyers;
- Facebook is a social media app and website which creates a platform for users to connect with one another; in a sense, most Facebook users are both producers of content (by posting things to be shared) and consumers of content (by reading others' posts);
- Google is a search engine which connects consumers of information with creators of online content;
- Apple and Microsoft both implement operating systems on mobile (iOS, Windows Phone) and computer (OS X, Windows); these operating systems are platforms which connect app creators with app users.

The recent and rapid growth in online content, combined with more advanced search algorithms and faster internet speeds, has increased the value of platforms. With so much information out there, a consumer can benefit from a centralized location for searching for consumer goods (Amazon, craigslist), food services (OpenTable, Yelp), rides (Uber, Lyft), streaming music (Spotify, Apple Music), or online video content (YouTube, Netflix).

Since in their role as multisided platforms, these firms play a facilitating role in the market, their behavior is quite different from that of a traditional input-to-output firm. These differences in behavior imply that we will need a unique set of tools to understand how platforms operate. For example, commonly, there is a "seller" and a "buyer" of the transaction the matchmaker facilitates - think Amazon or Spotify. And almost always, the multisided platform will charge different prices to the different sides of the market. In a seeming paradox, there is often one side of the market which receives a subsidy to participate, paying a price below cost, or even paying no price at all! For Amazon, it is the buyer who is subsidized: Amazon sellers must pay to post an item for sale. For Spotify, it is the seller who is subsidized by receiving payments per listen, while buyers of the musical content are charged subscription fees.

In this chapter, we will construct the basic building blocks of a model of multisided platforms. In doing so, we will uncover the unique conditions under which a platform can survive and thrive, and how platforms' behavior compares and contrasts with the behavior of a traditional input-to-output firm.

Platforms ... and more?

If you are observing carefully, many of the examples of platforms above also engage in some input-to-output business. Besides developing operating systems, for example, Apple also produces and sells physical phones, computers, and accessories. Amazon produces eReaders, hands-free voice-controlled speakers, and original TV programming. Netflix produces original content too. How does this behavior interact with their roles as platforms?

The focus of this chapter will be on firms' platform behavior. However, when we have a few theoretical tools under our belt, we will return to this question, to see how Netflix original content and Apple devices can either substitute for or complement how the firms operate as platforms.

1 Features of multisided platforms

To best understand how platforms operate, there are several essential features we need to understand. When combined, these features create a unique space for multisided platforms.

1.1 Interconnected demand

The services of a platform are demanded by consumers on each side of the market. Let's use OpenTable as our example. The services of OpenTable are demanded by both diners (the buyers? side of the market) and restaurants (the sellers? side of the market.) Therefore, it is as if OpenTable is demanded by two groups: buyers and sellers.

This may appear to resemble a firm who engages in third-degree price discrimination[4], where the firm sells a good or service to different groups of consumers at different prices. As we will see, however, the platform differs in a key way.

In short, the platform *does not sell the same service to each group*. Diners do not buy meals from OpenTable, but use the platform to more easily find restaurants from whom to buy. Therefore, what OpenTable sells to diners is both its platform and its slate of restaurants to potential diners! Likewise, it sells its platform and access to its pool of potential consumers to restaurants who sign up[5]. A user on one side of the market ultimately seeks the platform insofar as the platform helps to facilitate that match or connection with a party on the other side of the market. In this way, demands on each side of the market are interconnected.

When buyers and sellers match, a transaction occurs. Not every home buyer is willing to purchase every home on the market, and not every seller is willing to sell to every buyer. The platform then generates revenue off of the transactions which occur[6]. Unlike a traditional firm, the platform must balance the demand characteristics (preferences, price sensitivity) of multiple contingents to make the platform work. A music streaming service would not survive very long if they only had artists whose music listeners didn't like!

1.2 Network externalities

The interconnectedness of demand forces the platform to rely on **positive network externalities**. You may know that a positive externality is an additional benefit which is generated from economic activity and which is outside of the initiating user. When you get a flu shot, for example, this generates a positive externality because on top of benefiting you, your protection against the flu will make others around you less likely to get the flu. A positive network externality is a positive externality which is generated due to each users' contribution to the size of the overall network

[4]**third-degree price discrimination**: firm pricing behavior wherein the firm sells a good or service to different groups of consumers at different prices; examples include student and senior citizen discounts.

[5]According to their website (https://restaurant.opentable.com/products/guestcenter), this is a network of 25 million diners.

[6]There can be different models of this, for sure. A real estate agent may only make her commission once a sale has been completed; however, Amazon may earn revenue from a seller posting an item for sale, regardless of whether that sale actually takes place.

of users. When you download a social media app, there is an immediate benefit you receive; but this download also generates a positive network externality by expanding the size of the network of users, making the app more desirable for others.

The role of positive network externalities is critical for platforms precisely because of interconnected demand. In order for OpenTable to be successful, restaurants need to sign up. But restaurants value OpenTable only insofar as there are diners who use its service - and diners only value OpenTable provided there is a wide selection of restaurants who participate. Like any firm who produces a network externality good, a platform seeks to reach a critical mass of users and start a positive feedback loop of growth: get enough restaurants so the number of diners grows, which will encourage growth from restaurants. There are many different strategies which can be used to kickstart this growth: be the first platform in the market (this is often referred to as first-mover advantage); focus on major players, such as inviting major recording artists to your streaming service; or create a particularly innovative or user-friendly platform to lower the costs for new users to join.

These network externalities function both across sides of the market (buyers need a selection of sellers and sellers need a large pool of buyers) and within sides of the market. As a potential diner, having more users on OpenTable is generally good, since as the network grows, it becomes more attractive for additional restaurants. However, this can backfire and lead to increased competition for the reservations desired by each individual diner. Think of a seller on Amazon, who may simultaneously welcome additional sellers to the site (which attracts more potential consumers) and express concern that additional sellers will increase competition and diminish profit.

1.3 Pricing: money side versus subsidy side

Since they need to take advantage of the network externalities across sides, platforms typically charge one side of the market a very low, zero, or negative price - the **subsidy side**[7] of the market. This ensures enough users on this side to attract demand on the other side of the market - the **money side**[8] - which is charged a higher price. For example, OpenTable does not collect a fee from diners for signing up, but they do charge restaurants for access to their platform.

[7] **subsidy side**: the side of the market which is charged a lower price, relative to the money side

[8] **money side**: the side of the market which is charged a higher price, relative to the subsidy side

The determination of which side is money and which side is subsidy is a key one for the platform. In some models, the sellers' side is money (OpenTable, Amazon), while in others, the buyers' side is charged the higher price (streaming music apps charge listeners subscription fees while subsidizing artists.) Generally, though, **price elasticity of demand**[9] can help the platform to plan its pricing strategy.

Recall that the price elasticity of demand measures buyers' sensitivity to changes in price. Relatively *elastic* demand characterizes consumers who are more sensitive to changes in price, while relatively *inelastic* demand characterizes consumers who are less sensitive to changes in price. As a general rule, *platforms tend to subsidize the side of the market where demand is more elastic, while charging a higher price on the side where demand is more inelastic.*

Interconnected demand and positive network externalities tell the story: the platform needs users on both sides of the market to participate in order to create transactions. But how to incentivize diners to use a site with no restaurants on it? Or incentivize restaurants to post openings on a site with no customers? The platform will often use the subsidy side of the market to charge a sufficiently low price to grow and maintain the number of users on that side, making the platform more attractive to users on the money side.

A platform still incurs a cost to facilitate transactions: developing and maintaining a website or app requires technological and human capital investment. Consider, for a moment, the platform's **marginal cost**[10] per transaction: the added cost from facilitating an additional transaction. In previous sections, we have seen how firm pricing relates to the firm's cost:

- perfectly competitive firms are able to charge a price *equal to their marginal cost*;
- firms with market power (monopolists, for example) are able to charge a price *above their*

[9]**price elasticity of demand**: the relationship between the percentage change in quantity demanded given a percentage change in price; formally, it can be computed as

$$\epsilon_D = \frac{\%\Delta Q_D}{\%\Delta P} = \frac{\Delta Q_D}{\Delta P}\frac{P}{Q_D} = \frac{dQ_D}{dP}\frac{P}{Q_D}$$

ϵ_D values are negative; large negative values ($\epsilon_D < -1$) indicate relatively elastic demand, while small negative values ($\epsilon_D > -1$) indicate relatively inelastic demand.

[10]**marginal cost**: the added cost incurred by the firm for an additional transaction; formally:

$$MC = \frac{\Delta Totalcost}{\Delta transaction} = \frac{\Delta TC}{\Delta Q} = \frac{dTC}{dQ}$$

marginal cost.

Platforms adhere to a different pricing strategy. The platform will charge a price *above its marginal cost* to the money side and charge a price *below its marginal cost* to the subsidy side. This behavior runs counter to other models of firm pricing. If there is a cost to running the platform, how can the platform offer it to users for free? Or even pay users to participate?

Let's use Spotify to develop the intuition. Listeners are the money side: paid subscribers fork over $10 per month for access to the service; artists are the subsidy: they receive payments from Spotify whenever a track of theirs is listened to. Musical artists demand Spotify, but they are charged a negative price. But Spotify *needs those artists* in order to attract listeners! So, naturally, Spotify may need to pay artists to participate. It's the interconnected demand story again: by attracting enough listeners, Spotify can cover their costs - and their payments to artists - through subscription fees.

In the next section, we will use a theoretical model of platform profit maximization to see if our pricing story is supported!

1.4 Bonus: Two-sided platforms versus multi-sided platforms and the freemium model

As we have discussed, two-sided platforms connect two groups of users, such as buyers and sellers. But more generally, platforms are described as multisided because they can feasibly connect *more than two* sides of a given market. It may seem counterintuitive - what would the third side be?

The most common example of the third side of the market is advertising. This is particularly relevant to online content and social media platforms. Consider Facebook. If the sellers (creators of content) and buyers (consumers of content) are both able to access the platform for free, how does Facebook generate revenue? Where is the money side? Advertisers are Facebook's money side, since they demand the platform for its "eyeballs": the views from users on either side. If advertisers aim to get their content in front of as many eyeballs as possible, they are willing to pay the platform for access to those eyeballs.

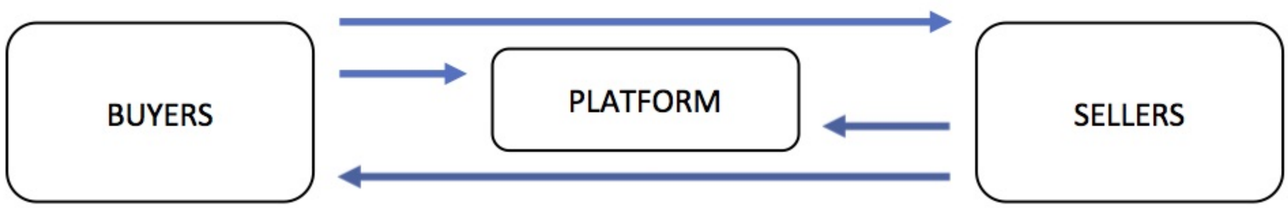

Figure 1: Two-sided platform. Buyers and sellers both demand the platform and in doing so demand the other side of the market.

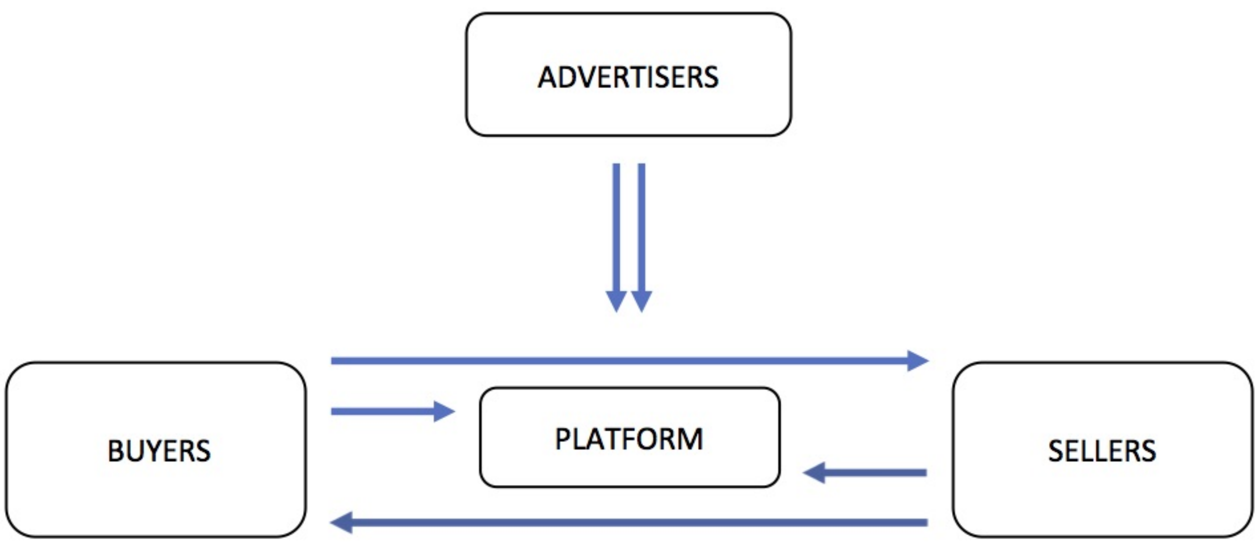

Figure 2: Multisided platform. Buyers and sellers both demand the platform. Advertisers are the third side of the market, demanding the platform's eyeballs for their advertising content.

Advertisers can be an critical money side for platforms like YouTube and Spotify. For most YouTube users, for example, content creation is free (sellers' side) and viewing is free (buyers' side), but YouTube will force ads on viewers and earn revenue from advertisers. Interestingly, YouTube also offers paid subscriptions - such as YouTube Premium or YouTube Music - for viewers who want an ad-free experience (among other perks). This is often referred to as the **freemium model**[11] of pricing. It is a common pricing model for music streaming apps, Amazon (free to use but Amazon Prime exists), and Facebook (content creators can pay extra to boost posts.) The freemium model can be best understood through the multisided model:

- Free YouTube account: buyers are a subsidy side, and advertisers are the money side;
- Paid YouTube Premium account with no ads: buyers do not have to watch ads but become a money side.

In a way, the buyers' side is part subsidy and part money. Users whose willingness to pay is lower sign up for the free version, are forced to see ads, and the platform earns revenue from advertisers. Users whose willingness to pay is higher sign up for the premium version, and therefore become an additional money side for the platform. With more than two sides to the platform, there are additional configurations for money sides and subsidy sides, and the platform can in effect price discriminate to extract revenue from an otherwise subsidy side of the market.

2 Two-sided platforms: a theoretical model

We introduce the multisided platform firm model here. In this benchmark multisided platform model, the platform is two-sided and the firm is a monopolist.[12] The multisided platform here will be in the business of producing *transactions*, denoted by Q. Both buyers (B) and sellers (S) will demand these transactions via the platform. For each transaction, the platform incurs a per-unit cost of $c > 0$. This is the unit cost of Uber in facilitating a driver-passenger connection, the cost of Amazon posting and linking a sale, or the cost of OpenTable to fill a seat at a restaurant. The

[11] **freemium model**: a model of pricing where users have the option for either a free version of a service or a paid version of the service with added features.

[12] More technical research drops these assumptions here, but maintaining them allows us to understand the main mechanisms of the platform more easily.

platform can charge one price to the buyer side of the market, P^B, and a different price to the seller side of the market, P^S.

Buyers demand transactions according to the buyers' demand function, $q^B(p^B)$, while sellers demand transactions according to the sellers' demand function, $q^S(p^S)$. Rochet and Tirole (2003) calls these individual demands *quasi-demand functions*. For simplicity here, we will model the number of actual transactions as the product of the (assumed independent) demand functions from each side of the market.[13] That is,

$$Q(p^B, P^S) = q^B q^S$$

The platform makes P^B per transaction from each buyer, P^S per transaction from each seller, and incurs cost of c per transaction. Therefore, the (monopolist) platform maximizes profit

$$\pi(P^B, P^S) = P^B Q + P^S Q - cQ = (P^B + P^S - c) q^B q^S$$

Demand in percentage terms

We will use demand functions in the model below, and typically, demand functions give the number of units demanded at a given price. For example, if $Q_D = 200 - P$, then at a price of 40, there are 160 units demanded. At a price of 0, there are 200 units demanded.

We will need a minor but important re-interpretation of demand in our model. Instead of measuring a quantity as a number of units, we will measure quantity as the *percentage of all possible units demanded*. For example, in the demand function above, the maximum number of units which could be demanded is 200, since this is the number of units demanded when the good is free! Therefore, it is equivalent to make the following two statements:

- At a price of 40, there are 160 units demanded;
- At a price of 40, 80% of the market is demanded;

[13]From Rochet and Tirole, quasi-demands are probabilistic, with a distribution of benefit b^i to each side of the market, $i = B, S$. If $q^B(P^B) = Pr(b^B \geq p^B)$ and $q^S(P^S) = Pr(b^S \geq p^S)$, then as long as benefits are independently distributed, a transaction occurs $q^S q^B$ of the time. This assumption is not required, but is used here for simplicity.

To emphasize this interpretation, we will write demand functions in small terms. For example, we will see demand written like

$$Q_D = 1 - \frac{1}{4}P$$

Therefore, at a price of 3, $Q_D = 0.25$. This is not saying that there is one fourth of a unit demanded, but, rather, that $0.25 = 25\%$ of the market is demanded at the price of 3. Do not be thrown off by the decimals as quantities demanded - they are percentages!

2.1 Linear case (symmetric)

As a first pass, let us solve a linear demand version of the model. Let $q^B = 1 - P^B$ and $q^S = 1 - P^S$. This gives platform profit as

$$\pi(P^B, P^S) = (P^B + P^S - c)(1 - P^B)(1 - P^S)$$

How does the platform simultaneously choose the pair of prices (P^B, P^S) to maximize its profit? Optimization dictates that the optimal prices must satisfy the following two equations[14]:

$$q^B - (p^B + p^S - c) = 0$$

and

$$q^S - (p^B + p^S - c) = 0$$

Substituting in quasi-demands and solving this system of equations yields optimal prices as $P^{B*} = P^{S*} = \frac{c+1}{3}$. If, for example, the unit cost is $c = \frac{1}{2}$, then $P^{B*} = P^{S*} = \frac{1}{2}$, $q^B = q^S = \frac{1}{2}$, and

[14] Taking partial derivatives with respect to prices yields

$$\frac{\partial \pi}{\partial P^B} = q^B q^S - q^S(p^B + p^S - c) = 0$$

and

$$\frac{\partial \pi}{\partial P^S} = q^B q^S - q^B(p^B + p^S - c) = 0$$

That is, the optimal pricing conditions are derived from standard multivariate optimization techniques. This section can be analyzed with or without knowledge of the underlying techniques.

$Q^* = \frac{1}{4}$. Given identical demands, the platform charges identical prices to each side of the market.

2.2 Linear case (asymmetric)

What if demands differ? Let $q^B = 1 - 2P^B$ and $q^S = 1 - P^S$. If we look at these two demand functions, the buyers' quasi-demand function is flatter than the sellers' quasi-demand function. This allows us to formulate a **hypothesis** based on our earlier discussion: given the differences across sides of the market, we expect the buyers' side to be the subsidy side and the sellers' side to be the money side.

Now the profit maximizing platform chooses prices to maximize

$$\pi(P^B, P^S) = (P^B + P^S - c)(1 - 2P^B)(1 - P^S)$$

Here, the optimal pricing conditions are given as[15]:

$$q^B - 2(p^B + p^S - c) = 0$$

and

$$q^S - (p^B + p^S - c) = 0$$

Substituting in quasi-demands here yields the system

$$1 - 4P^B - 2P^S + 2c = 0$$

and

$$1 - 2P^S - P^B + c = 0$$

[15] Taking partial derivatives with respect to prices yields

$$\frac{\partial\pi}{\partial P^B} = q^B q^S - 2q^S(p^B + p^S - c) = 0$$

and

$$\frac{\partial\pi}{\partial P^S} = q^B q^S - q^B(p^B + p^S - c) = 0$$

This gives optimal prices as $P^{B*} = \frac{c}{3}$ and $P^{S*} = \frac{c}{3} + \frac{1}{2}$. With $c = \frac{1}{2}$, we see that $P^{B*} = \frac{1}{6}$ and $P^{S*} = \frac{2}{3}$. Then, $q^B = \frac{2}{3}$, $q^S = \frac{1}{3}$, and $Q^* = \frac{2}{9}$. Given different demands, different optimal prices are offered to different sides of the market. Notice that since the per unit cost is $\frac{1}{2}$, the sellers' side of the market is charged a price *above marginal cost*, while the buyers' side of the market is charged a price *below marginal cost*! The buyers' side in effect **subsidizes** the sellers' side and our hypothesis is confirmed.

Platforms ... and more? Continued.

We have seen how platforms maximize profit by subsidizing one side of the market and marking up prices on the other.

3 Platform price markups

3.1 Reviewing the price markup and Lerner Index

Recall that a monopolist facing linear demand operates on the *relatively elastic* part of the demand curve, where $\epsilon_D < -1$. The degree of elasticity determines how drastically the monopolist can increase its price relative to its marginal cost. A perfectly competitive firm, on the other hand, charges a price exactly equal to its marginal cost! We can derive a formula for the monopolist's **price markup**, defined as $\frac{P}{MC}$, the ratio of the price it charges to its marginal cost. Using the product rule to differentiate the monopolist's total revenue function $TR(Q) = P(Q)Q$, then using the monopolist's profit maximizing condition that $MR = MC$ at Q^*, it can be shown that

$$\frac{P}{MC} = \frac{1}{1 + \frac{1}{\epsilon_D}}$$

Notice that for a perfectly competitive firm who faces an (infinitely elastic) demand curve, the price markup is essential 1, i.e. no markup. Again, a monopolist will operate on the elastic part of a linear demand curve.[16] For very elastic demand, the price markup is high, while for moderately elastic demand, the price markup is still greater than one but smaller in absolute value.

[16]This holds provided the monopolist's marginal cost is positive. This occurs because for the monopolist to maximize profit, it must choose output where MR = MC, and only for the elastic portion of linear demand is the monopolist's marginal revenue positive.

The **Lerner Index** serves as another measure of monopolists' market power, defined as

$$\frac{P - MC}{P} = -\frac{1}{\epsilon_D}$$

The Lerner Index takes values from 0 (no market power, infinitely elastic demand) to 1 (maximum market power, the most inelastic but still relatively elastic demand, such as $\epsilon_D = -1.1$).

3.2 Elasticity and platform pricing: a conundrum?

What is the relationship between the **price elasticity of (quasi-)demand from buyers**, the **price elasticity of (quasi-)demand from sellers**, and the optimal prices chosen by the platform? Since the individual demand functions are independent, we can compute price elasticities in the standard way as

$$\epsilon_B = \frac{dq_B}{dP^B}\frac{P^B}{q^B} \text{ and } \epsilon_S = \frac{dq_S}{dP^S}\frac{P^S}{q^S}$$

When evaluated at equilibrium, they give the elasticities of buyers' and sellers' demands. In the symmetric linear demand case, recall that $MC = c = \frac{1}{2}$, and that $P^{B*} = P^{S*} = \frac{1}{2}$. On the individual demands, therefore, there is no price markup! Upon looking at the individual price elasticities, we can calculate that $\epsilon_B = \epsilon_S = -1$. Since individual demands are unit elastic at these prices, this is difficult to reconcile with the monopolist case described above because a monopolist must operate on the elastic part of the demand curve.[17]

Now, consider the asymmetric demands case. Recall that the buyers have flatter (more relatively elastic) demand in general, while sellers have steeper (more relatively inelastic) demand in general. But what happens when we compute elasticities at the optimal prices? Interestingly, when we examine the different demands case, where $c = \frac{1}{2}$, $P^{B*} = \frac{1}{6}$, and $P^{S*} = \frac{2}{3}$, something odd seems to show up. With $q^B = \frac{2}{3}$, the price elasticity of (quasi-)demand from buyers is given by $\epsilon_B = -\frac{1}{2}$; demand from buyers is relatively inelastic, and buyers are charged a **lower** price! Conversely, given $q^S = \frac{1}{3}$, the price elasticity of (quasi-)demand from sellers is $\epsilon_S = -2$: sellers' demand is relatively elastic and sellers are charged a higher price! This runs counter to our intuitive hypothesis from earlier, which seemed to be confirmed. What's going on here?

[17] The actual price markup measure leads to an undefined result when using $\epsilon_D = -1$.

An explanation can be found by going back to the original monopolist's problem. Recall that the monopolist operates on the elastic part of the demand curve, and as a result, always has at least some (however small) price markup. And indeed, as we see here, the monopolist charges a markup to the demanders (here, the seller side of the market) whose demand is elastic! What about the buyers? Their demand is **inelastic** at the monopolist's optimal prices: consequentially, the monopolist gives the buyers side of the market not a price markup, but a **price markdown**!

On the most straightforward level, this should make sense. Any successful transaction must include buyers, but if $c = \frac{1}{2}$, and the monopolist attempts to even charge a price *equal to its marginal cost*, the quantity demanded from buyers ($q^B = 1 - 2P^B$) would be zero! So, to get even one buyer on board, the monopolist must mark down to the buyers side of the market.

But let's take a step back, and assume the marginal cost is just c. In the different demands case, $P^{B*} = \frac{c}{3}$ and $P^{S*} = \frac{c}{3} + \frac{1}{2}$. From here, we can see that $q^B = 1 - \frac{2}{3}c$ and $q^S = \frac{1}{2} - \frac{1}{3}c$. Or, rearranged,

$$q^B = 2(\frac{3-2c}{6}) \text{ and } q^S = \frac{3-2c}{6}$$

The buyers' side will always have double the quantity demand of the sellers' side, regardless of the value of c. On the price elasticities of (quasi-)demand,

$$\epsilon_B = -2\frac{\frac{c}{3}}{2(\frac{3-2c}{6})} = -\frac{2c}{3-2c}$$

and

$$\epsilon_S = -\frac{\frac{c}{3} + \frac{1}{2}}{\frac{3-2c}{6}} = -\frac{2c+3}{3-2c}$$

First, notice that when $c = \frac{1}{2}$, we get the numerical elasticities above. Most importantly, though, sellers' demand will *always* be more elastic than buyers' demand at optimal platform pricing! And sellers will *always* face a higher price than buyers! So, this counterintuitive anomaly - that the more inelastic side of the market gets charged the lower price - is more pervasive than just our simple numerical example.

3.3 An adjusted Lerner index for platforms

It turns out that this phenomenon is not the result of an ironclad pricing rule, but, rather, entirely by design! To see why, first let's transform the general profit maximization problem for the platform by taking the log:

$$\log \pi = \log\left(P^B + P^S - c\right) + \log\left(q^B\right) + \log\left(q^S\right)$$

Solving this profit maximization problem is identical to solving the original one, so we can differentiate to get first order conditions:

$$\frac{d \log \pi}{dP^B} = \frac{1}{P^B + P^S - c} + \frac{1}{q^B}\frac{dq^B}{dP^B} = 0$$

and

$$\frac{d \log \pi}{dP^S} = \frac{1}{P^B + P^S - c} + \frac{1}{q^S}\frac{dq^S}{dP^S} = 0$$

Let's tackle the buyers' side first. If we multiply both sides by P^B, we get that

$$\frac{P^B}{P^B + P^S - c} = -\epsilon_B$$

where the right-hand side contains the price elasticity of demand from the buyers' side of the market. Now flipping both sides, we see

$$\frac{P^B + P^S - c}{P^B} = -\frac{1}{\epsilon_B}$$

which we can slightly rearrange to

$$\frac{P^B - (c - P^S)}{P^B} = -\frac{1}{\epsilon_B}$$

This is essentially the Lerner Index on the buyers' side of the market - with one minor change! It gives the platform's markup on buyers relative to the platform's *opportunity cost of a price increase on the buyers' side.* The loss of a transaction due to an increase in buyers' price P^B has opportunity cost $c - P^S$ because the cost of the transaction c is partially offset by the P^S collected from the sellers' side of the market. Similarly, it can be shown that under optimal platform pricing, it must be true that

$$\frac{P^S - (c - P^B)}{P^S} = -\frac{1}{\epsilon_S}$$

On both sides of the market, relative to the true opportunity cost of a price increase, the standard Lerner Index relationship between price markup and price elasticity of demand holds!

In our different demands example, with $P^B = \frac{1}{6}$, $P^S = \frac{2}{3}$, and $c = \frac{1}{2}$, let's interpret. On the sellers' side, the opportunity cost of an increase in P^S is $c - P^B = \frac{1}{3}$, and the proportion of the price charged to sellers which is markup above opportunity cost is exactly one half - consistent with a price elasticity of demand for sellers of -2. On the buyers' side, since the platform will more than cover the marginal cost of the transaction with the price it collects from sellers, the opportunity cost of an increase in P^B is *negative*: $c - P^S = -\frac{1}{6}$.[18] The difference between the buyers' price and the opportunity cost of an increase in that price doubles the price itself, and links to the price elasticity of demand for buyers value of $-\frac{1}{2}$.

3.4 Pricing conundrum solved

So, why does this confusion occur? Does the platform's pricing strategy adhere to our intuitive understanding of subsidizing the elastic side and marking up the inelastic side?

In short, we *can* say that our intuition is confirmed.

There are what one might call *feedback effects* between price and price elasticity of demand. There is a common conception of demand elasticity relating to the slope of a linear demand function: flatter curves more elastic, steeper ones more inelastic. And, generally, it is true that for a monopolist platform facing two different demand curves, given a constant $P = P^B + P^S$, it generates more revenue to charge a lower price to the side with the flatter curve and a higher price to the side with the steeper curve.

But, upon further examination, we know that along the same linear demand function, higher prices correspond to higher values of ϵ_D, while lower prices correspond to lower values of ϵ_D. That is, elasticity changes along different points on the same linear demand curve! Sometimes, the

[18]Negative opportunity cost implies that instead of giving something up when P^B is increased, the platform gains something when it increases P^B!

calculation of an exact value of ϵ_D at a given point along a demand curve is referred to as a *point elasticity* for precisely this reason.

Therefore, when the platform implements a pattern of optimal pricing which lowers the price to the side with the flatter demand curve, it slides downward along that curve, pushing the value of ϵ_D to be more inealstic! Similarly, the platform increases the price to the side with the steeper curve, and in doing so, moves ϵ_D to be more relatively elastic.

That is, the monopolist is still increasing the price on the side of the market "conventionally understood" to be more inelastic and decreasing the price on the side of the market understood to be more elastic. What we observe is the price on the inelastic side of the market driven high enough to render demand relatively elastic, and the price on the elastic side of the market driven low enough to render demand relatively inelastic. This result is well examined by Krueger (2009).

References

[1] Krueger, M. (2009). The elasticity pricing rule for two-sided markets: A note. *Review of Network Economics*, 8(3).

[2] Rochet, J. C., & Tirole, J. (2003). Platform competition in two?sided markets. *Journal of the european economic association*, 1(4), 990-1029.

[3] Rochet, J. C., & Tirole, J. (2006). Two?sided markets: a progress report. *The RAND journal of economics*, 37(3), 645-667.