Chapter 9:
Auction Theory

This chapter will explore the topic of Auction Theory, the forms of auctions, their properties, and applications. We will examine auctions from both the perspective of the bidder (optimal bidding strategies) and the perspective of the seller (how to design an auction to maximize revenue generated).

Further, we will go more in depth into the many real-world applications of auction theory, the fundamental types of auctions and their properties, how truthful bidding arises in a second-price sealed-bid auction, and why auction design matters.

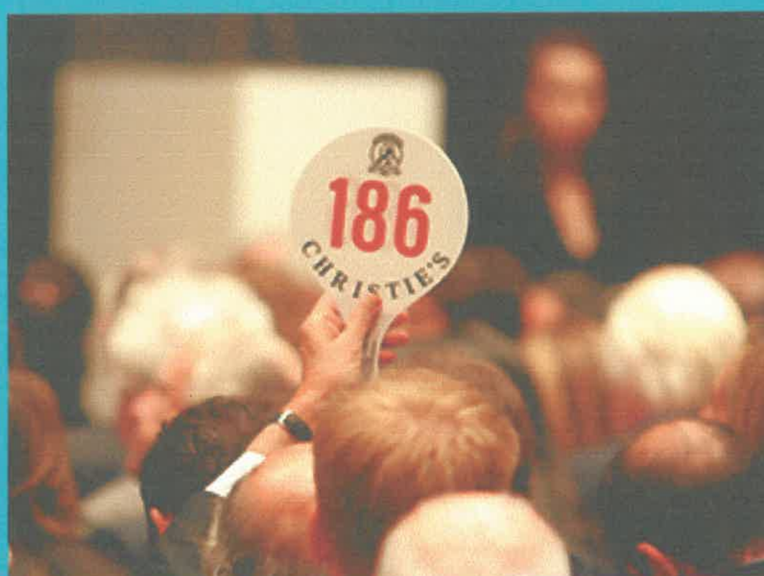
What you will see in this chapter:

- Auction properties
- Exploration of the four main types of auctions
- Auction applications
- Perspective of the bidder
- Perspective of the seller

Ascending-Bid Auctions

What is the first scenario that pops into your head when you hear the word "auction?" Chances are, it's something along the lines of the Ascending-bid Auction or *English Auction*, perhaps the most commonly recognized form of auction today. In this type of auction, the auctioneer (the individual in charge of starts the bidding process for an item at a certain value. The auctioneer then gradually raises the price as bidders either vocally or electronically place their respective bids. This process continues until there is only one bidder left: the highest bidder. That bidder receives the good or service at the winning price.

Real World Example: Christie's Auction House



A bidder places his offer at Christie's Auction House in London.

Founded in 1766, Christie's has served as the world's premier auction house for many years. Offering roughly 450 auctions in over 80 different categories annually, it currently possesses the world's largest market share with approximately $3.5 billion in sales in the first half of 2012 representing the highest total in company and art market history.

During a normal auction at Christie's a group of individual, registered bidders gather to place their individual bids on the item at hand. For this example, let's use a traditional Grecian Urn as the item for sale. (In this situation, the auctioneer will be in charge of gradually increasing the price as opposed to the actual seller). The auctioneer starts the bidding at £100,000. Bidder #50 raises her paddle to signify her bid, in other her willingness to pay £100,000 for the Urn. The auctioneer rebutts by increasing the value to £150,000. Bidder #22 raises his paddle. Both bidder #50 and bidder #22 along with other individuals want the urn, but each has a *maximum* amount that they are willing to pay which is unknown to all other bidders. This pattern continues on, until finally bidder #50 outbids her competitors and wins the Grecian Urn with the highest bid of £350,000.

Descending-Bid Auctions

One auction type that is highly related to the Ascending-bid Auction is the Descending-bid Auction or *Dutch Auction*. You can think of it as the Ascending-bid Auction in reverse. This means that instead of starting at some low price and gradually increasing over time, the auctioneer starts the bidding at a high initial value and works down. The auctioneer lists off gradually decreasing values until the moment when the first bidder places a single bid. The bidding stops after this point and the first bidder pays the price for the item. Dutch Auctions are often used as a method for price setting (see the Example below) as well as a way to sell items.

Real-World Example:

- Price setting company shares at an Initial Public Offering (IPO)

Example:

"Em Coe." is a new up-and-coming business that would like to start selling shares, or pieces of the company, to prospective investors. However, the company owners do not know the optimal price at which to sell these shares. Rather than go to an investment bank to calculate and compute this optimal price based on investor interest, Em Coe. decides to use the more efficient process of Dutch Auctions.

Since these type of auctions are usually conducted online, potential investors in Em Coe. must first obtain the necessary access codes and bidder information to enter the auction. The auctioneer, called the underwriter in this scenario, begins the bidding process with a prohibitively high value per share. Let's say Wynn, our underwriter, starts at $50/share. No bids are offered. He gradually lowers it to $45, where 2 bids are offered for 100,000 shares. The next value is $42, where 4 bids come in for 400,000 shares. This process continues until the underwriter lowers the price to $33 and all shares (2,300,000) are sold.

The table below illustrates Em Coe.'s dutch auction:

Price	Bids	Shares	Cumulative Shares
$50	0	0	0
$45	2	200,000	100,000
$40	4	800,000	1,000,000
$35	10	1,000,000	2,000,000
$33	3	300,000	2,300,000

When the auction closes, Wynn analyzes the results. The highest number of bids adding up to 1,00,000 shares sold at a price of $35. Thus, $35 appears to be the optimal price per Em Coe. share.

Conclusion:

As the real world example illustrates, Dutch Auctions can be a more efficient way of determining prices. The U.S. Treasury also uses this method to sell Treasury securities, and many U.S. companies use it for share buybacks.

Fun Fact!
Dutch Auctions originated in the Netherlands as a way for farmers to sell flowers at open markets. Hence the name, *Dutch Auction.*

First-Price Sealed-Bid Auctions

HA.

One of the sneakiest forms of auctions is the first-price sealed-bid auction! In the example of this type of auction, bidders simultaneously and independently submit offers and the item is awarded to the highest bidder who must pay their bid. The catch here however, is that players place their bids sealed, *individually* to the auctioneer. They don't know how much others are bidding! Traditionally, these bids were sealed in envelopes and opened simultaneously by the auctioneer.

First-price auctions are among the most common forms of auctions.

Example: Sophia bids $290 and Emma bids $350 for the same marble statue of a macadamia nut (has to be between 0-900). Neither player is aware of what the other is bidding. Emma is awarded the painting at a price of $350 (V=valuation of the bidder). In this situation; Emma's payoff would be v-350 and Sophia's payoff would be 0.

painting or statue?

Note that in the example of a first-price auction, neither player has any reason to bid more than they value the item, because this would result in a negative payoff for either individual.

It actually would be more beneficial for players to bid on the item *less* than they value it to obtain a positive payoff in the possibility that (s)he wins the auction! This action can be called "shading," such as "shading" a little bit off of your value in order to receive a positive payoff.

How much should you "shade" off of your bid?
That can be dependent on two factors.

1. If you decide to bid extremely close to your value, you run the risk of not getting much positive payoff if you happen to win the bid.
2. But if you lower your bid too low to receive a higher payoff, you run the risk of not submitting the highest bid and losing the bet!

What balance of these tradeoffs would result in the best outcome for the individual?

Balancing these tradeoffs mostly depends upon the knowledge of the other bidders and their distribution of their individual values?

Keeping all other bidding properties the same, In an example of a situation in which there is a large amount of bidders in one first-price sealed-bid auction, it is more likely for the competing bids to be higher, therefore one would want to bid higher in order to place the highest bid. Alternatively, in the situation in which one is bidding in a first-price sealed-bid auction where are fewer players, it would be smart to bid lower, as there is less of a chance of others outbidding you.

Nash Equilibrium:

Individual Strategies: Let us assume that Sophia and Emma individually decide to strategize! They both decide to bid a *fraction* of their actual valuation of the item, (a). In this example, Sophia and Emma's actual bid will be similar to this b=av (v is the valuation of the bidder, a=fraction of the valuation of the bidder).

If we assume that Sophia uses this strategy of b=av, we should find Emma's optimal strategy. Lets say Emma's valuation is v, and she is considering a bid of x. If she wins this auction against Sophia, then her payoff will be v-x. In this case, Emma would prefer to have x be relatively small. *As x is lowered, however, it is less likely or Emma to actually win the auction.* Emma only wins if Sophia's bid falls below amount x of Emma's bid.

Because Sophia is bidding according to her function of b=av (a being a fraction of the valuation and v being the valuation), a bid of x would be made by Sophia if her valuation were x/a.

In the case of which Sophia had a valuation below this (x/a), she would bid less than x.

(uniformly?)

Because Sophia's value is distributed formally between 0 and 900, the probability that Sophia's value will be greater than x/a is x/900a.

Therefore, if Emma decides to bid at value x, she can expect to win the bet at a probability of x/900a for the marble macadamia nut statue!

If Emma bids x, her expected payoff is going to be equal to:

The probability of winning times the surplus she receives if she wins

i.e. $\frac{(v-x)x}{900a}$

In order to solve for Emma's optimal bid, we must take the derivative with respect to x and set it equal to 0, in which we should get x=v/2. ← Here we see that Emma's best response to Sophia's strategy is to bid exactly have of her valuation! This is Emma's optimal strategy!

Interesting example. wish it were a little more clear.

The equilibrium of a first-price auction is efficient: Here the player with the highest valuation of the good will win the auction. Here we can see there is a Nash equilibrium because the best strategy for Emma is what we predicted from the beginning. The bidding parameter turns out to be a=½, therefore, the bid values for both Emma and Sophia equal b(v)=v/2 respectively.

4 key facts to understanding First-Price Auctions:

(1). There is a tradeoff between the probability of winning and the surplus obtained by winning. (2). it is optimal to bid less than one's valuation.
(3). auctions can be designed to induce "truthful" bidding.
(4). competitive bidding produces information about the bidders' valuations and can allow the seller to extract surplus from the trade.

Second-Price Auctions

In this auction format, bidders simultaneously and independently submit bids b_1 and b_2. The item is then awarded to the highest bidder at a price equal to the **second-highest** bid.

For example, if bidder 1 bids $100 for a painting and bidder 2 bids $75 for the painting then bidder 1 will win the painting but pay only $75. The payoff in this scenario for each bidder would be v_1-75 for player 1 (v_1 being bidder 1's valuation of the painting) and 0 for bidder 2 (because bidder 2 doesn't get the painting, but also doesn't have to pay for the painting).

Real-World Examples:
- eBay (essentially, see below)
- Pricing mechanism that search engines use to sell keyword-based advertising is a generalization of the second-price auction

To find the **Nash equilibrium** of second-price auctions we must first understand the dominant strategy for each bidder. The dominant strategy for every bidder v with independent, private values is to bid your true value. In other words, **the best choice of bid is exactly what the object is worth to you.**

Understanding the Second-Price Auction

Define in terms of bidders, strategies, and payoffs
- *i*=bidder
- *v*=bidder's true value for the object
- *b*=bid

Strategy: Bidder *i*'s strategy is an amount *b* to bid as a function of their true value *v*.

Payoff to bidder *i* with value *v* and bid *b:*

3 different outcomes:
1. If *b* is not the winning bid then payoff to *i* is 0
2. If *b* is winning the bid and some other b_j is the second-place bid, then payoff to *i* is v_i-b_j
3. In the event of a tie, we assume that the bid that was placed first is the "winning bid" however, in this situation payoff will be 0 because in the event of a tie the first-place and second-place bids are equal

Claim: In a sealed-bid second-price auction it is a dominant strategy for each bidder to choose a bid b_i=v_i
- To prove this claim, and understand the Nash equilibrium outcome of a second-price auction, we need to show that if bidder *i* bids b_i=v_i then no deviation from this bid would improve their payoff regardless of what strategy everyone is using

Prove: Suppose that bidder i is considering whether to bid $b_i=v_i$ or to bid some other amount $b_i=x$

- Ex. $x>v_i$
  1. Bidder j's bid b_j is at lease as large as x
     a. Bidder i will lose the auction regardless of whether they bid x or v_i
  2. Bidder j's bid is between v_i and x
     a. Bidder i does worse bidding x than by bidding v_i and if they bid v_i then they lose the auction and get payoff of 0, if they bid x then they win the auction and have to pay b_j which will give them a negative payoff of $b_j=v_j$
  3. Bidder j's bid b_j is less than v_i
     a. Bidding v_i and bidding x yield the same payoffs to bidder i

*The same can be proved for $x<v_i$

Good.

Conclusions:

1. The dominant strategy in a second-price auction is for each bidder to bid their true value (this strategy will lead to Nash equilibrium)
2. The equilibrium of a second-price auction is efficient because the object goes to the player with the highest valuation
3. The seller is able to locate the highest valuation bidder, but the seller is not able to appropriate all of the surplus of the trade because the winning bidder pays only the second-highest bid (except in the event of a tie)
4. The seller's expected revenue equals the expected second-highest valuation

Real World Example [eBay] explained:

excellent example.

eBay is not what you would typically think of as a second-price sealed-bid auction, however there is a feature referred to as a **proxy bid** that eBay utilizes in their online auctions. eBay works like a typical ascending-bid auction in that users can bid on an item for a set amount of time and then the last bidder to submit the higher bid wins the item. The proxy bid allows for users to put in their "true value" of the product and then eBay will automatically keep upping their bid in response to other bidders bids until they either win the item or the bids surpass their true value.

Ex. Sophia wants to buy some new shoes and finds the most amazing pair of Crocs on eBay. Sophia would be willing to pay $25 for the Crocs, meaning her true value for the Crocs v=25. The current highest bid for the shoes is at $5 so Sophia doesn't need to place a bid at her true value. She places a bid for $5.50 (we are assuming that $0.50 is the minimum required for a new bid) and then walks away. When she checks back an hour later the current bid is $7. Sophia doesn't have time to sit in front of the computer for the last two hours of the auction so she sets her proxy bid at $25 and eBay will keep upping any bids that beat hers until her maximum is reached (max=v).

Understanding the Second-Price Auction

So far, we have assumed that the seller in different auctions must sell the object. However, it is possible for the seller to keep the item. In this situation, the seller is able to place what is called a reserve price on the good or service being sold. This is the minimum price at which the seller is willing to sell the item. Let's consider how the seller's expected payoff will change with the presence of a reserve price.

Say the seller values the item at $u \geq 0$. Notice that this is the payoff he or she gets from keeping the item rather than selling it. If $u>0$ it is best not to use a simple first-price or second-price auction as the winning bid could be less than u in either case and the seller would not want to sell the item. Instead, it is preferable for the seller to announce the reserve price of *r* before holding the auction even if $u=0$.

If it is true that a seller should declare a reserve price, what should the value of *r* be? If the item is worth u to the seller, he or she should obviously price $r \geq 0$. However, the reserve price that maximizes the seller's revenue is r strictly greater than 0 ($r>0$). Let's see why this is true.

<u>No reserve price:</u>

- Second-price auction with a single bidder
- Bidder value-uniformly distributed at [1,0]
- Seller value: $u=0$

→ item is sold to the bidder at 0.

<u>Reserve price, $r>0$:</u>

- probability $1-r$

→ bidder's value *above r*

→ item is sold to the bidder at *r*.

- probability of *r*

→ bidder's value *below r*

→ seller keeps the item, with a payoff of 0.

The seller's expected revenue is $r(1-r)$ which is maximized at $r=½$ with $u=0$. If $u>0$, the expected revenue is $r(1-r) + ru$. Therefore, a seller's optimal *r* is halfway between u and the maximum possible bidder price for a second-price auction with a single bidder.

Bidder Valuations

Private values: each bidder knows their own valuation of the object but not that of the other bidder; the bidders usually have different valuations

Example: Sale of a Treasury bill in the United States

- This type of bill pays a stated amount of money after a specific length of time
- The value of a T-bill depends on future interest rates and on the risk of default
  - Different bidders may have different ideas about the future of interest rates

Example: Sale of distressed or seized property to commercial resellers

- Winning buyer will carefully evaluate the property (after the sale), recondition it, and sell it to consumers
  - Value of doing this is independent to the buyer
    - Bidders may get different signals about the valuation of the property before making bids (one bidder better at spotting damage, another may be better at estimating repair costs)

Common-value setting: bidders' valuations are the same, but no one has perfectly accurate information

Example: 2 bidders with the same valuation of the item being auctioned:

- Bidder 1 and Bidder 2
- Valuation=Y
  - $Y=y_1+y_2$
  - y_1 and y_2 are uniformly distributed

In the first-price sealed bid auction, each bidder will respond to individual signals. Bidder 1 observes signal y_1 and bidder 2 observes signal y_2:

- Scenario 1: Bidder 1 bids expected valuation of the object given his signal y_1.
  - If bidder 1 bids his expected valuation and wins the payoff will not be zero because if he wins the auction he learns something about the other bidder's signals

The "Winner's Curse":
A bidder wins when the other bidders bid less. However, this implies that the other bidders must have received signals that indicated the value of the item was less than what the winning bidder bid (they received a relatively bad sign on the auction).

- **Strategic implication:** one should factor in that winning yields information and this information should be used in formulating expected valuation

Conclusion

As we learned in this chapter, different types of auctions yield different perspectives, strategies, and expected returns. We see auctions everyday whether that is the procurement of seized land by the government, eBay, live auction houses, etc. Economists continue to be intrigued by auctions as a part of Game Theory. Some auction forms like the second-price auction format are not especially common in the real world but serves as an important example of why bidders would bid a certain way. It is important to understand the differences in auction design as either a buyer or a seller and how these differences will impact their outcomes. For example, in a second-price auction the seller is able to locate the highest valuation of their item but will not receive the surplus of the trade because the winning bidder only has to pay the second-highest bid. Being aware of these things will only help you (as either the buyer or the seller) to come out with the best end of the deal.

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