Volatility, Financial Markets and The Minority Game

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Why is Volatility Important?

• High Frequency Trading has sped up financial markets
• As a result markets are more complex than ever. This has lead to worries about volatility.
• This can be seen in the Flash Crash of 2010
• Need a model to study volatility
Empirical Results of HFT

• HFT is between 27% (Benos and Sagade) and 73% (Hendershott) of all trades

• Two types
  – Passive Liquidity Supplying Market Makers
  – Aggressive Liquidity Demanders

• HFT Decrease volatility, increase liquidity help bring prices closer to the theoretical correct price.
The Minority Game

• 2k+1 Agents
• Two actions 1 and -1
• Agents are rewarded if they are on the minority side
• No a-priori best action
  – Agents will have difficulty forming correct beliefs
  – Agents act inductively
• Bounded Rationality
  – Agents must adjust their actions
The Minority Game and Volatility

• Social Optimum Occurs when the variance around the mean is minimized.
• Many actions in financial markets depend on a degree of balance.
Nash Equilibria

• If exactly k+1 agents are playing one action then that configuration is a Nash equilibrium.
• If every agents chooses their action with even probability then the action profile is a Nash equilibrium.
• An infinite number of other Nash Equilibrium with some agents playing pure strategies and other agents are playing mixed strategies.
Learning in the Minority Game

• A strategy is a rule which prescribes an action for a given history.
• Agents have a memory of length M. Agents use the history of the past M rounds to determine their actions.
• Agents randomly draw s strategies.
• Each strategy has a score which increases when the agent does well using it.
• This learning model is hard to analyze analytically.
An example of a Strategy

<table>
<thead>
<tr>
<th>History</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,1,1</td>
<td>1</td>
</tr>
<tr>
<td>1,1,-1</td>
<td>1</td>
</tr>
<tr>
<td>1,-1,1</td>
<td>-1</td>
</tr>
<tr>
<td>1, -1, -1</td>
<td>1</td>
</tr>
<tr>
<td>-1, 1, 1</td>
<td>-1</td>
</tr>
<tr>
<td>-1, 1, -1</td>
<td>1</td>
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<tr>
<td>-1, -1, 1</td>
<td>1</td>
</tr>
<tr>
<td>-1,-1,-1</td>
<td>-1</td>
</tr>
</tbody>
</table>
Numerical Simulations and Alpha

- Alpha = $2^m/2k+1$ is the most important parameter in the minority game
- Intuitively, Alpha is a measure of the crowdedness of the state space
- Crowds/anti-crowds
Other Learning Models

• Fictitious Play
  – Normal fictitious play will converge to the equal probability mixed strategy but will alternate
  – Stochastic fictitious play will converge as below

• Replicator Dynamics/Reinforcement Learning
  – Actions which result in good consequences are used more and actions which result in bad consequences are used less
  – Pure Strategy NE stable mixed Strategy NE Unstable
HFT as Longer Memory

- One way to model HFT is to allow some traders to have longer memory lengths
- This would represent the ability of HFT to process information more quickly than traditional investors
- Johnson et al. numerically simulated this case and found that mixed populations resulted in less volatility.
Johnson et al. (1999)
HFT as last mover advantage

• Another way to change the minority game into an extensive form game.

• If the proportion of HFT is small then if HFT moved last some volatility could be eliminated around the equilibrium
Conclusion

• Strategic Heterogeneity is Important
• Strategic Heterogeneity provides an additional way HFT can decrease volatility.
• Other aspects of HFT such as positional externalities, adverse selection and fairness need to be considered